Appendix

The Discipline Panel in Mathematics recognizes that each campus, and the mathematics department within, is unique having a different educational philosophy, mission, and values. We do not believe that we can, nor that we should, prescribe the way mathematics is taught nor the examples that are used for an assessment. The purpose of this appendix is two-fold. First, this appendix serves to illustrate one or two possible examples that can be used to assess each learning outcome. The mathematical content of each question will vary depending each campus' definition of general education and its specific plan for assessing strengthened campus-based assessment. Secondly, this appendix can illustrate the way a grading rubric can be used to assess a student's response to the learning outcomes. Graders using this rubric should be able to place a student's response into the one of the four levels of meeting the standards. The idea is to achieve consistency and inter-rater reliability.

Sample illustrations of the learning outcomes

Key: Completely Correct (CC), Generally Correct (GC), Partially Correct (PC) and Incorrect (IC)

Learning Outcome #1: Students will demonstrate the ability to interpret and draw inferences from mathematical models such as formulas, graphs, tables, and schematics.

Example 1:

Perform the indicated hypothesis test. Write the null and alternative hypothesis. State whether to reject or fail to reject the null hypothesis and give a reason. Write a conclusion in the context of the problem.

A sample of 20 C-GCC students resulted in the following one-way commuting distances, in miles.

8	12	24	3	7
15	13	32	10	6
9	22	12	18	5
17	19	38	6	10

Use the given data to test the claim that the mean commuting distance for all C-GCC students is more than 12 miles. Use a 0.05 level of significance.

Level	Example	Comments
CC	Claim: $\mu > 12$ miles H_1 $\mu = 12$ miles H_0 p-value = 0.138 > 0.05 = α fail to reject H_0 Sample data do not support the claim that the mean commuting distance for all C-GCC students is greater than 12 miles.	This response addresses all three parts of the problem completely and accurately.
GC	$p > 12$ miles H_1 $p = 12$ miles H_0 p = 0.138 is greater than 0.05 fail to reject H_0 Sample data do not support the claim that the mean commuting distance for all C-GCC students is greater than 12 miles.	This response addresses all three parts to the problem but labels the null and alternative hypotheses with *p* instead of μ. Also the p-value is labeled using p.
PC	$\mu = 12$ H_1 $\mu < 12$ H_0 $\mu = 0.137 < 0.05$ reject The sample data support the claim that the commuting distance is greater than 12 miles.	This response demonstrates a lack of understanding of the problem. The null and alternative hypotheses are reversed, the p-value is labeled with μ and the conclusion is wrong.
IC	$\bar{x} = 14.3$	No clue.

Example 2:

A golfer hits a nine iron and the ball's height above the ground, in feet, is given by $h(x) = -0.03x^2 + 3x$, where x is the horizontal distance, in yards, from where the ball is hit.

What are the coordinates of the vertex of the graph of $y = h(x)$? What is the contextual significance of the coordinates of the vertex?

Level	Example	Comments
CC	Vertex: (50,75) The ball reaches its maximum height of 75 feet above the ground when it is 50 yards horizontally from where it was hit.	This response answers the two parts of the question completely and accurately.
GC	Vertex: (50,75) The ball is 75 feet above the ground.	This response answers the first part of the question correctly and addresses some aspect of the contextual significance of the vertex.
PC	Vertex: (50,75)	The vertex is correct but the student does not demonstrate an understanding of the contextual significance.
IC	Vertex: –50 The golfer missed the ball	There is no understanding of the problem demonstrated in this response.

Learning Outcome #2: Students will demonstrate the ability to represent mathematical information symbolically, visually, numerically and verbally.

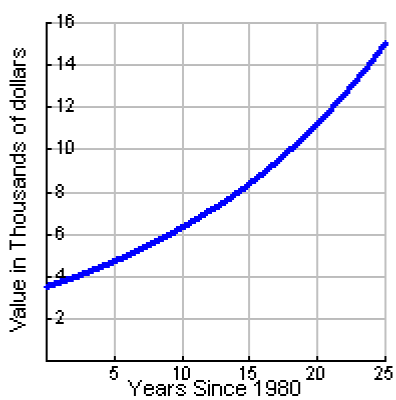
Example 1:

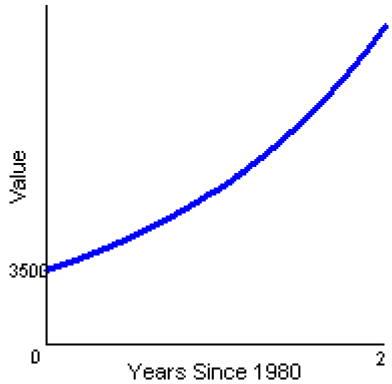
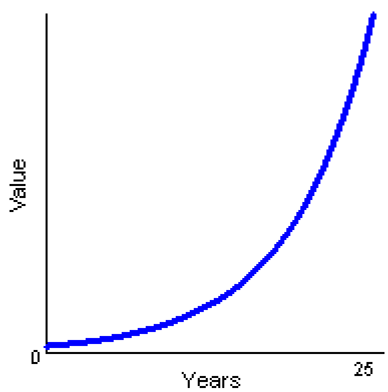
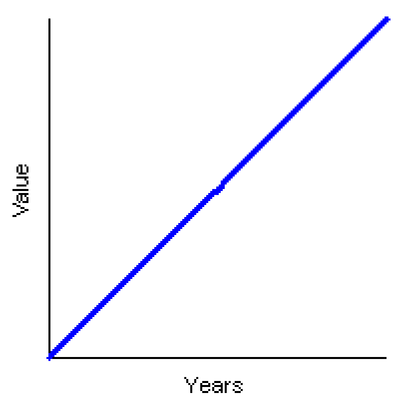
Note: *In the following examples a separate level of achievement is illustrated for each of four representations. This is for illustration purposes only. In specific cases it may be possible to award an overall achievement level in an assignment that requires more than one method of representation.*

1. In 1980, the value of an antique desk was $3500. The value of the desk has been increasing at a rate of 6% per year.

a) Write an equation that expresses the value of the desk in dollars as a function of the number of years since 1980. (*symbolic representation*)

b) Sketch a graph of this function for the period 1980 – 2005. Include labels and scales. (*visual representation*)

Level	Example	Comments
CC	a. $V = 3500(1.06)^n$, where V is the value of the desk in dollars and n is the number of years since 1980.  b.	Both the symbolic and visual representations are accurate and complete.
GC	a. $V = 35(1.06)^n$, V = value, n = years	a. Expression in dollars, rather than hundreds, was required. The representation is otherwise correct, so it is lacking in a minor way.

	b. 	b. Mistake in horizontal scale may be due to omitting the digit "5". Vertical scale could be better indicated with values above the intercept. Shape of curve is correct and labels are present.
PC	a. $V = 3500 + 1.06x$, $V = 3500(.06)^n$ b. 	a. Each of these equations misses a <u>major</u> element – the relation is exponential, and the growth factor is 1.06. Variables are not defined. Initial value is correct. b. Vertical scale is completely absent, intercept appears wrong. Horizontal label incomplete. Shape is correct.
IC	a. $V = 3500x + 6x$ b. 	a. Equation indicates neither initial value nor pattern of increase correctly. b. The partially correct labels and indication that the function is increasing do not demonstrate an understanding of how to represent the relation described in the question.

Example 2:

2. The linear equation $B = 60 + 0.40m$ expresses the amount of a monthly wireless telephone bill in dollars as a function of the number of minutes of calls (beyond the number of minutes included as part of basic service).

a) Construct a table of values for m and B in ten-minute increments from 0 to 50. (*numerical representation*)

b) Interpret the meaning of the vertical intercept and slope of this equation in terms of the variables in the question. (*verbal representation*)

<table>
<tr><th>Level</th><th>Example</th><th>Comments</th></tr>
<tr><td>CC</td><td><table><tr><td>m</td><td>B</td></tr><tr><td>0</td><td>60</td></tr><tr><td>10</td><td>64</td></tr><tr><td>20</td><td>68</td></tr><tr><td>30</td><td>72</td></tr><tr><td>40</td><td>76</td></tr><tr><td>50</td><td>80</td></tr></table>
b. The vertical intercept, 60, indicates that the monthly charge is $60 without including any charges for the number of minutes of calls. The slope, 0.40, indicates that there is a $0.40 (40 cent) charge per minute of calls beyond the included minutes.</td><td>a. This table is exactly what was required.

b. The explanation is complete and the language is unambiguous.</td></tr>
<tr><td>GC</td><td><table><tr><td>m</td><td>B</td></tr><tr><td>10</td><td>64</td></tr><tr><td>20</td><td>68</td></tr><tr><td>30</td><td>72</td></tr><tr><td>40</td><td>76</td></tr></table>
b. The vertical intercept, 60, is the starting point. The slope, 0.40, indicates that it is increasing by $0.40 per minute.</td><td>a. Display is correct, proper procedure was used to arrive at numerical values, but the table is incomplete for what was required.
b. The phrases “starting point” and “it is increasing” do not fully explain the meaning in the context of the variables in the question. A correct general understanding of vertical intercept and slope is indicated.</td></tr>
</table>

<table>
<tr>
<td>PC</td>
<td>
<table>
<tr><td>m</td><td>B</td></tr>
<tr><td>0</td><td>60</td></tr>
<tr><td>10</td><td>60.40</td></tr>
<tr><td>20</td><td>60.80</td></tr>
<tr><td>30</td><td>61.20</td></tr>
<tr><td>40</td><td>61.60</td></tr>
<tr><td>50</td><td>62</td></tr>
<tr><td colspan="2">or</td></tr>
</table>
<table>
<tr><td>m</td><td>B</td></tr>
<tr><td>0</td><td>60</td></tr>
<tr><td>10</td><td>100</td></tr>
<tr><td>20</td><td>140</td></tr>
<tr><td>30</td><td>180</td></tr>
<tr><td>40</td><td>220</td></tr>
<tr><td>50</td><td>260</td></tr>
</table>

b. The vertical intercept is 60. T[...]ll start $60. The slope is increasing by[...]
</td>
<td>
a. Incorrect application of slope to the values of the independent variable. In this context, the computational error would be a major flaw.

b. To s[...]at the slope is increas[...]by its own constan[...]lue suggests the student [...]mply (incorrectly) parroting a memorized phrase and does not demonstrate understanding in a specific context.
</td>
</tr>
</table>

<table>
<tr>
<td>IC</td>
<td>
<table>
<tr><td>m</td><td>B</td></tr>
<tr><td>0</td><td>60.40</td></tr>
<tr><td>1</td><td>120.80</td></tr>
<tr><td>2</td><td>181.20</td></tr>
<tr><td>3</td><td>241.60</td></tr>
<tr><td>4</td><td>302</td></tr>
<tr><td>5</td><td>362.40</td></tr>
</table>

b. The vertical intercept is 60 and the slope is increasing by 0.40.
</td>
<td>a. The procedure used to compute these values is completely incorrect.

b. Although the numbers are those of the question, there is no interpretation related to the information given, as required.</td>
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</table>

Learning Outcome #3: Students will demonstrate the ability to employ quantitative methods such as, arithmetic, algebra, geometry, or statistics to solve problems.

Example:

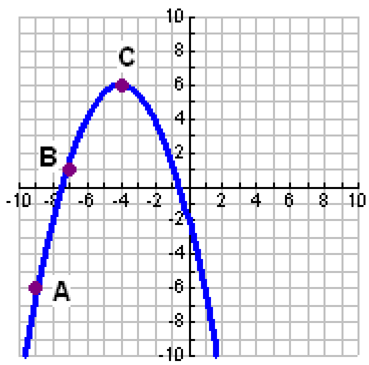
Solve for x: $5e^{2x-1} = 5000$

Level	Example	Comments
CC	$5e^{2x-1} = 5000$ $e^{2x-1} = 1000$ (2x-1) ln (e) = ln(1000) 2x-1 = ln (1000) 2x = ln(1000)+1 x = (ln(1000)+1) / 2	This solution completely and accurately arrives at the result. All steps are clearly displayed and the plan is evident and logical.
GC	$5e^{2x-1} = 5000$ $e^{2x-1} = 1000$ (2x-1) = ln(1000) 2x-1 = ln (1000) 2x = ln(1000)-1 x = (ln(1000)-1) / 2	The plan is evident and logical. The solution contains a minor arithmetic mistake (subtracting 1 from each side instead of adding 1).
PC	$5e^{2x-1} = 5000$ (2x-1) ln (5e) = ln(5000) 2x-1 = ln (5000) / ln(5e) $2x = \frac{\ln(5000)}{\ln(5e)} + 1$ $x = \frac{\frac{\ln(5000)}{\ln(5e)} + 1}{2}$	This solution attempts to use logarithms to solve the equation but does not demonstrate a true understanding of the properties of logarithms or that the base for the is e not 5e.
IC	$5e^{2x-1} = 5000$ $e^{2x-1} = 1000$ 2x-1 = 1000 2x=1001 x=500.5	This solution does not recognize that the equation is an exponential equation and therefore requires the use of logarithms for the solution. The student ignores the e.

Learning Outcome #4: Students will demonstrate the ability to estimate and check mathematical results for reasonableness

Example:

The graph of $f(x)$ is shown below. Rank $f'(A), f'(B)$ and $f'(C)$ from largest to smallest. Explain your reasoning.



Level	Example	Comments
CC	The derivative of a function at a point is the slope of the tangent line to the function at that point. The slope of the tangent line is positive at both points A and B and is 0 at point C. The tangent line at A is steeper, so the slope of the line at A is greater than the slope of the line at B. $f'(A) > f'(B) > f'(C)$	This solution correctly and accurately answers the question.
GC	The derivative of a function at a point is the slope of the tangent line to the function at that point. $f'(A) > f'(B) > f'(C)$	The student understood the concept but the explanation was slightly lacking.
PC	The derivative at C is 0. So it should be the smallest. $f'(A) > f'(B) > f'(C)$	Though the inequality is correct, the explanation does not indicate that the student has a clear understanding of the derivative.
IC	The y value for C is the largest. The y value for B > A. $f'(A) < f'(B) < f'(C)$	The student does not understand the idea of derivative.

Learning Outcome #5: Students will demonstrate the ability to recognize the limits of mathematical and statistical methods.

Example:

Learning Outcome # 5 would probably best be imbedded in other problems since the learning outcome really deals with student reflection. As such learning outcome # 5 is addressed in part d.

In 1965, an anthropologist used the exponential model $P(t) = 3{,}347{,}361{,}927\, e^{.02053\,t}$ to model the world's population at time t (years) with $t = 0$ representing the year 1965.

a. Complete the following table.

Year	Predicted population	Actual population	Percent error
1965		3,347,361,927	
1966		3,417,544,528	
1967		3,487,234,405	
1968		3,559,028,982	
1969		3,633,608,846	
1970		3,708,751,360	

b. Assuming that a good prediction is anything within 1% of the actual population then was the model a good predictor for the six years in question?

c. Given that the world population in 2000 was 6,081,527,896, was the model a good predictor of the population in that year? Explain.

d. What, if anything, can be said of the assumptions made in the above model and the effect of those assumptions on the validity of the model. Is this a reasonable model for population growth?

Level	Example	Comments
CC	a. Complete the following table. Predicted population Percent error 3,347,361,927 0 % 3416793544.2546 -0.02197 %	This is perceived as completely correct since in part d the student was able to ascertain which assumptions did not affect the correctness of the model and which ones affected the model over a longer period of time

	3487665325.3096 0.01236 % 3560007142.3164 0.02748 % 3633849487.0401 0.00662 % 3709223483.7118 0.01273 % b.Yes c. 2000-1965 = 35 $P(35) = 6866967186.0643$ The percent error is $\frac{6866967186.0643 - 6081527896}{6081527896} = 12.915\ \%$. This model was not a good predictor since it produced an error bigger than 1 %. d. It was assumed that the population grew continuously which gave non-whole number answers but this should not really effect the validity if one rounds off answers to the nearest whole number and remembers that these are estimates. - It was assumed that the model was an exponential model with a growth rate of 2.053% per year. This gave good answers during the first 6 years of predicting but did not predict well 35 years in the future. It is a good model as long as the growth rate stays pretty constant (in the short term) but is probably not a good predictor in the long term since the growth rate can fluctuate more.	A completely correct solution could have the predicted populations rounded off to the nearest whole number. This will not effect whether or not the number is a good predictor.
GC	a. Complete the following table. Predicted population Percent error	This was generally correct as the student was able to identify that the assumption of exponential growth

	3,347,361,927 0 % 3416793544.2546 -0.02197 % 3487665325.3096 0.01236 % 3560007142.3164 0.02748 % 3633849487.0401 0.00662 % 3709223483.7118 0.01273 % b. Yes c. 2000-1965 = 35 $P(35) = 6866967186.0643$ The percent error is $\frac{6866967186.0643 - 6081527896}{6081527896} = 12.915\ \%$. This model was not a good predictor since it produced an error bigger than 1 %. d. It was assumed that the model was an exponential model with a growth rate of 2.053% per year. This gave good answers during the first 6 years of predicting but did not predict well 35 years in the future. It is a good model as long as the growth rate stays pretty constant (in the short term) but is probably not a good predictor in the long term.	only worked well in the short run but did not identify that a discrete population was being modeled with a continuous function. In the case that a student makes a computational error, then the response in part d should be consistent with the findings in the previous part.
PC	a. Complete the following table. Predicted population Percent error 3,347,361,927 0 %	The student recognized when the model was a good predictor and when it was not but failed to provide any explanations as to why

	3416793544.2546 -0.02197 % 3487665325.3096 0.01236 % 3560007142.3164 0.02748 % 3633849487.0401 0.00662 % 3709223483.7118 0.01273 % b. Yes c. 2000-1965 = 35 $P(35) = 6866967186.0643$ The percent error is $\frac{6866967186.0643 - 6081527896}{6081527896} = 12.915\ \%$. This model was not a good predictor since it produced an error bigger than 1 %. d. The function gave good answers during the first 6 years of predicting but did not predict well 35 years in the future.	this happened.
IC	In part d. there is no explanation (rationale) of a response.	To receive any credit at all the student needs to be able to at least partially articulate why he/she gave a particular response.